

Measurement and Dimensional Analysis



Units of Measure

A quantitative observation, or measurement, ALWAYS consists of two parts: a *number* and a *unit*.

Two major measurements systems exist: English (US and some of Africa) and Metric (the rest of the globe!)

- SI system – 1960 an international agreement was reached to set up a system of units so scientists everywhere could better communicate measurements. Le Système International in French; all based upon or derived from the metric system

Table 1.1 The Fundamental SI Units

Physical Quantity	Name of Unit	Abbreviation
Mass	kilogram	kg
Length	meter	m
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Amount of substance	mole	mol
Luminous intensity	candela	cd

Table 1.3 Some Examples of Commonly Used Units

Length	A dime is 1 mm thick. A quarter is 2.5 cm in diameter. The average height of an adult man is 1.8 m.
Mass	A nickel has a mass of about 5 g. A 120-lb person has a mass of about 55 kg.
Volume	A 12-oz can of soda has a volume of about 360 mL.

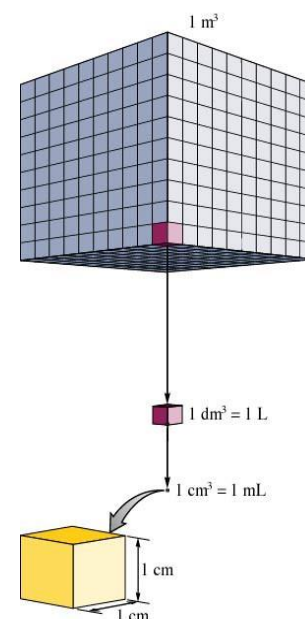
Table 1.2 The Prefixes Used in the SI System (Those most commonly encountered are shown in blue.)

Prefix	Symbol	Meaning	Exponential Notation*
exa	E	1,000,000,000,000,000,000	10^{18}
peta	P	1,000,000,000,000,000	10^{15}
tera	T	1,000,000,000,000	10^{12}
giga	G	1,000,000,000	10^9
mega	M	1,000,000	10^6
kilo	k	1,000	10^3
hecto	h	100	10^2
deka	da	10	10^1
—	—	1	10^0
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.000000000001	10^{-12}
femto	f	0.000000000000001	10^{-15}
atto	a	0.000000000000000001	10^{-18}

*See Appendix 1.1 if you need a review of exponential notation.

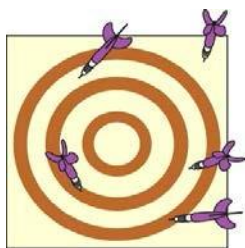
KNOW THE UNITS AND PREFIXES shown in **BLUE!!!**

- Volume – derived from length; consider a cube 1m on each edge $\therefore 1.0 \text{ m}^3$
 - A decimeter is 1/10 of a meter so
 $(1\text{m})^3 = (10 \text{ dm})^3 = 10^3 \text{ dm}^3 = 1,000 \text{ dm}^3$
 $1\text{dm}^3 = 1 \text{ liter (L)}$ and is slightly larger than a quart also
 $1\text{dm}^3 = 1 \text{ L} = (10 \text{ cm})^3 = 10^3 \text{ cm}^3 = 1,000 \text{ cm}^3 = 1,000 \text{ mL}$
 AND $1 \text{ cm}^3 = 1 \text{ mL} = 1 \text{ gram of H}_2\text{O}$ (at 4°C if you want to be picky)

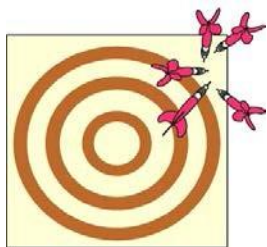


Precision and Accuracy

- **Accuracy** – correctness; agreement of a measurement with the true value
- **Precision** – reproducibility; degree of agreement among several measurements.
- **Random or indeterminate error** – equal probability of a measurement being high or low
- **Systematic or determinate error** – occurs in the same direction each time



(a)



(b)



(c)

The results of several dart throws show the difference between precise and accurate.

- (a) Neither nor precise (large random errors).
- (b) Precise but not accurate (small random errors, large systematic error).
- (c) Bull's-eye! Both precise and accurate (small random errors, no systematic error).

Exercise 1 Precision and Accuracy

To check the accuracy of a graduated cylinder, a student filled the cylinder to the 25-mL mark using water delivered from a buret and then read the volume delivered. Following are the results of five trials:

<i>Trial</i>	<i>Volume Shown by Graduated Cylinder</i>	<i>Volume Shown by the Buret</i>
1	25 mL	26.54 mL
2	25 mL	26.51 mL
3	25 mL	26.60 mL
4	25 mL	26.49 mL
5	25 mL	26.57 mL
<i>Average</i>	25 mL	26.54 mL

Is the graduated cylinder accurate?

Note that the average value measured using the buret is significantly different from 25 mL. Thus, this graduated cylinder is not very accurate. It produces a systematic error (in this case, the indicated result is low for each measurement).

Significant Figures and Calculations

Determining the Number of Significant Figures (or Digits) in a Measurement

- Nonzero digits are significant. (Easy enough to identify!)
- A zero is significant IF and ONLY IF it meets one of the conditions below:
 - The zero in question is “terminating AND right” of the decimal [must be both]
 - The zero in question is “sandwiched” between two significant figures
- Exact or counting numbers have an ∞ amount of significant figures as do fundamental constants (never to be confused with derived constants)

Exercise 2 Significant Figures (SF)

Give the number of significant figures for each of the following experimental results.

- A student’s extraction procedure on a sample of tea yields 0.0105 g of caffeine.
- A chemist records a mass of 0.050080 g in an analysis.
- In an experiment, a span of time is determined to be 8.050×10^{-3} s .

a. three; b. five; c. four

Reporting the Result of a Calculation to the Proper Number of Significant Figures

- When \times and \div , the term with the **least** number of *significant figures* (\therefore least accurate measurement) determines the number of **maximum** number of significant figures in the answer. (It’s helpful to underline the digits in the least significant number as a reminder.)

$$4.56 \times \underline{1.4} = 6.38 \xrightarrow{\text{corrected}} \underline{6.4}$$

- When $+$ and $-$, the term with the least number of *decimal places* (\therefore least accurate measurement) determines the number of significant figures in the final answer.

$$\begin{array}{r} 12.11 \\ 18.0 \quad \leftarrow \text{limiting term (only 1 decimal place)} \\ \underline{1.013} \\ 31.123 \xrightarrow{\text{corrected}} 31.1 \text{ (limits the overall answer to only one decimal place)} \end{array}$$

- pH – the *number of significant figures in least accurate measurement* determines *number decimal places* on the reported pH (usually explained in the appendix of your text)

Rounding Guidelines for the AP Exam and This Course:

- Round **ONLY** at the end of all calculations (keep the numbers in your calculator)
- Examine the significant figure one place beyond your desired number of significant figures. IF > 5 round up; < 5 drop the remaining digits.
- Don’t “double round”! Example: The number 7.348 rounded to 2 SF is reported as 7.3
In other words, **DO NOT** look beyond the 4 after the decimal and think that the 8 rounds the 4 up to a five which in turn makes the final answer 7.4.
[Even though you may have conned a teacher into rounding your final average this way before!]

Dimensional Analysis

Example: Consider a straight pin measuring 2.85 cm in length. Calculate its length in inches.

Start with a conversion factor such as $2.54 \text{ cm} = 1 \text{ inch}$ \therefore you can write TWO

Conversion factors: $\frac{1 \text{ in}}{2.54 \text{ cm}}$ or $\frac{2.54 \text{ cm}}{1 \text{ in}}$. Why is this legal? Both quantities

represent the exact same “thing” so the conversion factor is actually equal to “1”.

To convert the length of the pin from cm to inches, simply multiply your given quantity by a conversion factor you engineer so that it “cancels” the undesirable unit and places the desired unit where you want it. For our example, we want inches in the numerator so our numerical answer is not reported in reciprocal inches! Thus,

$$2.85 \cancel{\text{cm}} \times \frac{1 \text{ in}}{2.54 \cancel{\text{cm}}} = 1.12 \text{ in}$$

Let’s practice!

Table 1.4 English–Metric Equivalents

Length	1 m = 1.094 yd 2.54 cm = 1 in
Mass	1 kg = 2.205 lb 453.6 g = 1 lb
Volume	1 L = 1.06 qt 1 ft ³ = 28.32 L

Exercise 3

A pencil is 7.00 in. long. Calculate the length in centimeters?

17.8 cm

Exercise 4

You want to order a bicycle with a 25.5-in. frame, but the sizes in the catalog are given only in centimeters. What size should you order?

64.8 in

Exercise 5

A student has entered a 10.0-km run. How long is the run in miles?

We have kilometers, which we want to change to miles. We can do this by the following route:

kilometers \rightarrow meters \rightarrow yards \rightarrow miles

To proceed in this way, we need the following equivalence statements (conversion factors):

$$\begin{aligned} 1 \text{ km} &= 1000 \text{ m} \\ 1 \text{ m} &= 1.094 \text{ yd} \\ 1760 \text{ yd} &= 1 \text{ mi} \end{aligned}$$

6.22 mi

Exercise 6

The speed limit on many highways in the United States is 55 mi/h. What number would be posted if expressed in kilometers per hour?

88 km/h

Exercise 7

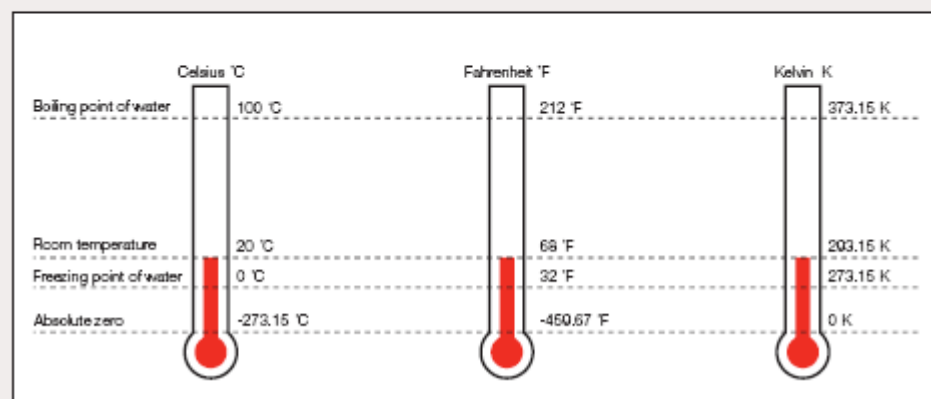
A Japanese car is advertised as having a fuel economy of 15 km/L. Convert this rating to miles per gallon.

35 mi/gal

Temperature

I suspect you are aware there are **three** temperature scales commonly in use today. A comparison follows:

Known Temperature				Required Temperature	Formulae
Celsius	°C	to	°F	Fahrenheit	$^{\circ}\text{F} = (1.8 \times ^{\circ}\text{C}) + 32$
Celsius	°C	to	K	Felvin	$\text{K} = ^{\circ}\text{C} + 273.15$
Fahrenheit	°F	to	°C	Celsius	$^{\circ}\text{C} = (^{\circ}\text{F} - 32)/1.8$
Fahrenheit	°F	to	K	Kelvin	$\text{K} = ^{\circ}\text{F} + 459.67/1.8$
Kelvin	°K	to	°C	Celsius	$^{\circ}\text{C} = \text{K} - 273.15$
Kelvin	°K	to	°F	Fahrenheit	$^{\circ}\text{F} = (1.8 \times \text{K}) - 459.67$



Notice a degree of temperature change on the Celsius scale represents the same quantity of change on the Kelvin scale.

Density

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

Exercise 8 Determining Density

A chemist, trying to identify the main component of a compact disc cleaning fluid, determines that 25.00 cm³ of the substance has a mass of 19.625 g at 20°C. Use the information in the table below to identify which substance may serve as the main component of the cleaning fluid. Justify your answer with a calculation.

<i>Compound</i>	Density (g/cm ³) at 20°C
Chloroform	1.492
Diethyl ether	0.714
Ethanol	0.789
Isopropyl alcohol	0.785
Toluene	0.867

Density = 0.7850 g / cm³ ∴ isopropyl alcohol